



A Coalgebraic Approach to the Y86 Processor Architecture

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- **Agenda**

- The Y86 Processor
- Algebras & Coalgebras
- Y86 Formal Model
- Y86 Simulator
- Conclusions

The Y86 Processor

- **A theoretic Processor based on IA32.**
- **Fewer Data Types, Instructions and Address Modes**
- **Still sufficiently complete to write almost any IA32 program**
- **Very good to understand the basis of a Processor Architecture**

Y86 Architecture

- **Instructions**

- Mov
- OPI – Integer operations
- jXX – Branch Conditions
- Call
- Ret
- Pushl
- Popl
- Halt

- **Registers**

- 8 Registers – numbered from 0 to 8
 - %eax %ecx %edx %ebx %esi %edi %esp %ebp

- **Condition Codes**

- ZF, SF, OF

Algebras & Coalgebras and Functors

- **Algebra**
 - Sets with pre-determined rules
 - Main issue is to Construct new Elements to include in the Algebra

$$F X = 1 + A * X * X$$

- **Coalgebra**
 - Main issue is to Destruct/Observe the Elements inside the Coalgebra
 - We are not concerned about the internal Set X

$$F X = 1 + A * X * X$$

Y86 Formal Model

- **Concepts to Formalize**
 - Y86 Instructions
 - Y86 State
 - Y86 Program
 - Y86 Functional Simulator

Y86 Instruction

An Instruction Is Just a State modifier

- $S \longrightarrow S$
 - $A \longrightarrow S^S$
 - $A_1 * A_2 * \dots * A_n \longrightarrow S^S$
- $$F X = G X \longrightarrow G X$$

Haskell definition

```
data Y86operation s v = Op (s -> (v, s))
```

Y86 State

- **We just formalize what we really need to know about...**

$$G X = A^8 = A * A * A * A * A * A * A * A$$

$$G X = A^8 * [F X]$$

Haskell definition

```
data Y86State = St { regs :: [Double],
                    mach :: [Y86operation Y86State ()]
                  }
```

Y86 Program

- A Y86 Program is a Sequence of Instructions

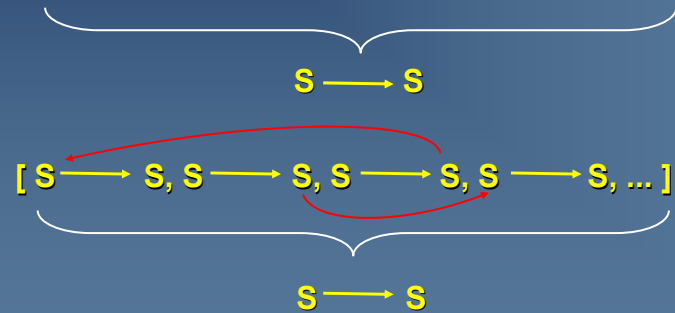
– $H X = 1 + F X * H X = [F X]$

Haskell definition

```
type Y86Program = [ Y86operation Y86State () ]
```

Y86 Functional Simulator

- Based on the Mathematical Concept of Monad

$$[S \longrightarrow S, S \longrightarrow S, S \longrightarrow S, S \longrightarrow S, \dots]$$


Haskell Simulator

```
instance Monad (Y86operation s)
  where return a      = Op (\x -> (a, x))
        (Op f) >>= g = Op (\s -> let (a, s2) = f s
                                   Op fun = g a
                                   in fun s2)
```

```
initSt = st (replicate 8 0)
```

```
prog2 :: [Y86operation Y86State ()]
prog2 = [opIRMovl 1.0 1, opIRMovl 5.0 2, opIRMovl 8.0 3,
         opAddl 1 2, opJne 4 2 3]
```

```
exec2 = snd (exec (sequence prog2) (initSt prog2))
```

```
Result = [1, 8, 8, 0, 0, 0, 0, 0]
```

Conclusions and Future Work

- Formal Methods is a precise tool to capture a Processor architecture.
- Other Mathematic approaches could have been used.
- Base model to start reasoning about the Y86 Processor. Several simplifications took place.
- Bissimulation relation between Y86 programs
- Y86 Program calculus
- Introduce weight in the instructions arrows